

COMPLETE SOLUTION AND MARKING SCHEME

Secondary Three Mathematics — Mock-Test (October 2018)

Section A - Multiple Choices (24 marks, 2 marks each)

For each of the following questions, please write down the corresponding alphabet (A, B, C or D) of your answer on P.2.

1. D
2. A
3. B
4. C
5. B
- *6. B (Canceled)
7. D
8. A
9. C
10. B
11. D
12. C

— The End of Section A —

Full Explanation to the Solutions of Section A.

$$\begin{aligned}
 1. \quad \frac{3ab}{2b+1} &= 1 - \frac{2a-3b}{b} \implies \frac{3ab}{2b+1} = \frac{b-2a+3b}{b} \\
 &\implies 3ab^2 = 8b^2 + 4b - 4ab - 2a \\
 &\implies a(3b^2 + 4b + 2) = 4b(2b+1) \\
 &\implies a = \frac{4b(2b+1)}{3b^2 + 4b + 2}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad 8x^2 - 6y^2 + 8xy + 2y - 4x &= (8x^2 + 8xy - 6y^2) + 2(y - 2x) \\
 &= (8x^2 + 8xy - 6y^2) + 2(y - 2x) \\
 &= (2x - y)(4x + 6y) - 2(2x - y) \\
 &= (2x - y)(4x + 6y - 2) \\
 &= 2(2x - y)(2x + 3y - 1)
 \end{aligned}$$

$$3. \quad \text{If } m = 2, n = 9 \text{ and } p = 3, \text{ then}$$

$$\begin{aligned}
 m\sqrt{n^p} + \frac{(pm^{n-p} - mn^m)^m}{5m^2p^2} &= 2\sqrt{9^3} + \frac{(3 \times 2^{9-3} - 2 \times 9^2)^2}{5 \times 2^2 \times 3^2} \\
 &= 2\sqrt{729} + \frac{(3 \times 2^6 - 2 \times 81)^2}{5 \times 4 \times 9} \\
 &= 2(27) + \frac{(192 - 162)^2}{180} \\
 &= 54 + \frac{(30)^2}{180} \\
 &= 59
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{27a^3 - 2}{3a(3a+b) + b^2} + \frac{b^3(a-2) - 2a + 4}{(2-a)(9a^2 + 3ab + b^2)} \\
 &= \frac{27a^3 - 2}{9a^2 + 3ab + b^2} + \frac{b^3(a-2) - 2(a-2)}{(2-a)(9a^2 + 3ab + b^2)} \\
 &= \frac{27a^3 - 2}{9a^2 + 3ab + b^2} - \frac{(a-2)(b^3 - 2)}{(a-2)(9a^2 + 3ab + b^2)} \\
 &= \frac{(3a)^3 - b^3}{9a^2 + 3ab + b^2} \\
 &= \frac{(3a-b)(9a^2 + 3ab + b^2)}{9a^2 + 3ab + b^2} \\
 &= 3a - b
 \end{aligned}$$

$$\begin{aligned}
5. \quad & A(2x-3)^2 + B(x+2)(x-2) + C \equiv 2x^2 + 3(B-2A)x + A - C \\
& \Rightarrow A(4x^2 - 12x + 9) + B(x^2 - 4) + C \equiv 2x^2 + 3(B-2A)x + A - C \\
& \Rightarrow (4A+B)x^2 - 12Ax + 9A - 4B + C \equiv 2x^2 + (3B-6A)x + A - C
\end{aligned}$$

By comparing the coefficients on both sides, we have

$$\Rightarrow \begin{cases} 4A + B = 2 & \text{-(Eq 1.)} \\ -12A = 3B - 6A & \text{-(Eq 2.)} \\ 9A - 4B + C = A - C & \text{-(Eq 3.)} \end{cases}$$

(Eq 1.) & (Eq 2.) implies $A = 1$ and $B = -2$.

Substitute $A = 1$ and $B = -2$ into (Eq 3.), we then obtain $C = -8$.

$$\text{Hence, } \frac{A}{B} - \frac{B}{C} = \frac{1}{-2} - \frac{-2}{-8} = -\frac{1}{2} - \frac{1}{4} = -\frac{3}{4}$$

6. (1) is correct: $a > b$ and $b > c$ implies $a > c$, then multiply -1 on both sides. We have $-a < -c$.

(2) is incorrect: From (1), we have proven that $a > c$. Consider the case that $a = 0$, $c = -1$
 $a^2 = 0$, $c^2 = 1$. Obviously in this case, $a^2 < c^2$.

(3) is correct: From (1), we have proven that $a > c$. Consider taking inverse of each side, we have $\frac{1}{a} < \frac{1}{c}$.

7. (1) is correct: If $a < 0$ and $b > 0$, we have $a < b$. Then multiply -1 on both sides. We have $-a > -b$.

(2) is correct: $a < 0$ means a is a negative number. $b > 0$ means b is a positive number. The product of a negative and a positive number must be negative i.e. < 0 .

(3) is correct: If $a < 0$ and $b > 0$, we have $a < b$. Subtract b by a , we have

$$b - a > 0 \Rightarrow \frac{1}{b-a} > 0$$

$$8. \quad \frac{-2x+5}{x+2} > 3 \Rightarrow -2x+5 > 3x+6$$

$$\Rightarrow -5x > 1$$

$$\Rightarrow x < -\frac{1}{5}$$

$$9. \quad \text{Set up the corresponding inequality: } \frac{3h(x+y)}{2} \leq 120$$

$$\Rightarrow h(x+y) \leq 80$$

$$\Rightarrow x+y \leq \frac{80}{h}$$

$$\Rightarrow x \leq \frac{80-hy}{h}$$

$$\Rightarrow \frac{1}{x} \geq \frac{h}{80-hy}$$

10. For simple interest,
Amount = $P + Pnr\%$. Put Amount = \$79200, $P = \$36000$. We have

$$\$79200 = \$36000 + \$36000 \times 8 \times \frac{r}{100}$$

$$\Rightarrow r = 15$$

11. The value of the rare crystal at the end of 2018 = $\$300000 \times (1 + 7\%) = \321000
The value of the rare crystal at the end of 2019 = $\$321000 \times (1 + 10\%) = \353100
The value of the rare crystal at the end of 2020 = $\$353100 \times (1 + 13\%) = \399003
The value of the rare crystal at the end of 2021 = $\$399003 \times (1 + 16\%) = \$462843.48 = \$462843$

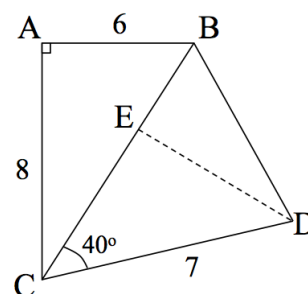
12. By Pythagoras's Theorem, we have $BC^2 = AB^2 + AC^2$

$$\Rightarrow BC = \sqrt{6^2 + 8^2} = 10 \text{ units}$$
Draw an altitude of BC through D , denotes the intersect point on BC by E .
We have $CE = 7 \cos 40^\circ$

$$\Rightarrow BE = BC - CE = 10 - 7 \cos 40^\circ$$
And, $ED = 7 \sin 40^\circ$
Hence, by Pythagoras's Theorem, we have

$$BD = \sqrt{ED^2 + BE^2} = \sqrt{(7 \sin 40^\circ)^2 + (10 - 7 \cos 40^\circ)^2}$$

$$\Rightarrow BD \approx 6.46 \text{ units}$$



Section B - Short Questions (31 marks)

1. Solution

$$\frac{(x^2y^{-3})^{-2}z}{x^{-2}(z^{-1}y)^3} = \frac{x^{-4}y^6z}{x^{-2}z^{-3}y^3} \quad (1M)$$

$$= x^{-4-(-2)}y^{6-3}z^{1-(-3)} \quad (1M)$$

$$= x^{-2}y^3z^4$$

$$= \frac{y^3z^4}{x^2} \quad (1A)$$

2. Solution

$$\begin{aligned} \text{LHS} &= \frac{1}{2(1 - \tan \theta)} - \frac{1}{2(1 + \tan \theta)} \\ &= \frac{1 + \tan \theta - (1 - \tan \theta)}{2(1 - \tan \theta)(1 + \tan \theta)} \\ &= \frac{2 \tan \theta}{2(1 - \tan^2 \theta)} \quad (1M) \end{aligned}$$

$$\begin{aligned} &= \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} \\ &= \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta - \sin^2 \theta} \quad (1M) \end{aligned}$$

$$\begin{aligned} &= \frac{\sin \theta \cos \theta}{1 - 2 \sin^2 \theta} \\ &= \text{RHS} \quad (1A) \end{aligned}$$

3. Solution

(a) $2x^2 + 3x - 14 = (x - 2)(2x + 7)$

(b) $2(x^2 - 2)^2 + 3(x^2 - 2) - x^2 - 10 = [2(x^2 - 2)^2 + 3(x^2 - 2) - 14] - x^2 - 10 + 14$
 $= [(x^2 - 2) - 2][2(x^2 - 2) + 7] - (x^2 - 4)$ (1M)
 $= (x^2 - 4)(2x^2 + 3) - (x^2 - 4)$
 $= (x^2 - 4)(2x^2 + 2)$ (1M)
 $= 2(x + 2)(x - 2)(x^2 + 1)$ (1A)

Alternative Method

$2(x^2 - 2)^2 + 3(x^2 - 2) - x^2 - 10 = 2x^4 - 8x^2 + 8 + 3x^2 - 6 - x^2 - 10$ (1M)
 $= 2x^4 - 6x^2 - 8$
 $= 2(x^2)^2 - 6x^2 - 8$
 $= (2x^2 + 2)(x^2 - 4)$ (1M)
 $= 2(x + 2)(x - 2)(x^2 + 1)$ (1A)

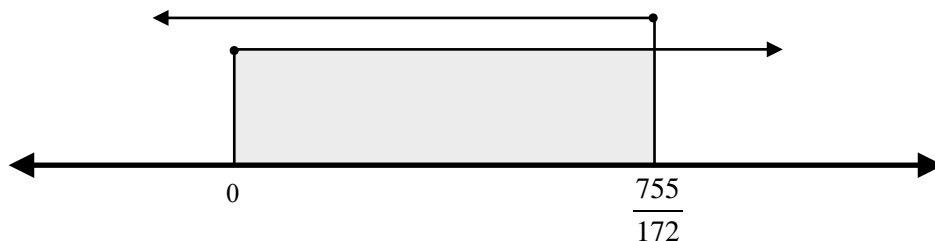
4. Solution

(a) $\frac{4}{3}(2x - 7) \leq \frac{-1}{5}x + \frac{13}{4}$ and $x \geq 0$
 $\Rightarrow 80(2x - 7) \leq -12x + 195$ and $x \geq 0$
 $\Rightarrow 172x \leq 755$ and $x \geq 0$
 $\Rightarrow x \leq \frac{755}{172}$ and $x \geq 0$ (1A)

$\Rightarrow 0 \leq x \leq \frac{755}{172}$ (OR $0 \leq x < 4.39$ correct to 3 sig. fig.) (1A)

(1A for Correct Graphical Solution)

(b) $0, 1, 2, 3, 4$. (1A)



5. Solution

$$4^p = 32^{2q} = 256^{3r} \Rightarrow 2^{2p} = 2^{5(2q)} = 2^{8(3r)} \quad (1M)$$

$$\Rightarrow 2p = 10q = 24r$$

$$\Rightarrow 2p = 10q \text{ \& } 10q = 24r \quad (1M)$$

$$\Rightarrow p : q = 5 : 1 \text{ \& } q : r = 12 : 5$$

$$\Rightarrow p : q : r = 60 : 12 : 5 \quad (1A)$$

$$\text{Therefore, } \frac{25q^2}{r(p-2r)} = \frac{25 \times 12^2}{(5)(60 - 2 \times 5)}$$

$$= \frac{72}{5} \quad (\text{OR } 14.4) \quad (1A)$$

6. Solution

$$\begin{aligned} \text{(a)} \quad \frac{u^4 - 16}{u^2 + 4} - 2u(u - 2) &= \frac{u^4 - 2^4}{u^2 + 2^2} - 2u(u - 2) \\ &= \frac{(u^2)^2 - (2^2)^2}{u^2 + 2^2} - 2u(u - 2) \\ &= \frac{(u^2 + 2^2)(u^2 - 2^2)}{u^2 + 2^2} - 2u(u - 2) \end{aligned} \quad (1M)$$

$$\begin{aligned} &= (u^2 - 2^2) - 2u(u - 2) \\ &= (u + 2)(u - 2) - 2u(u - 2) \end{aligned} \quad (1M)$$

$$\begin{aligned} &= (u - 2)(u + 2 - 2u) \\ &= (u - 2)(2 - u) \\ &= -(u - 2)^2 \end{aligned} \quad (1A)$$

$$\text{(b)} \quad 2u(u - 2) - \frac{u^4 - 16}{u^2 + 4} = 36 \Rightarrow \frac{u^4 - 16}{u^2 + 4} - 2u(u - 2) = -36$$

$$\Rightarrow -(u - 2)^2 = -36 \quad (1M \text{ for Using Part a})$$

$$\Rightarrow (u - 2)^2 = 36$$

$$\Rightarrow (u - 2) = 6 \text{ or } (u - 2) = -6$$

$$\Rightarrow u = 8 \text{ or } u = -4 \quad (1A)$$

7. Solution

$$\text{Cost} = 840 \times \$2.4 = \$2016$$

(1A for Both Correct)

$$\text{Income} = 840 \times (1 - 22.5\%) \times \$x = \$651x$$

$$\text{Profit Percentage} \geq 36\%$$

$$\Rightarrow \frac{\$651x - \$2016}{\$2016} \geq 36\% \quad (1M)$$

$$\Rightarrow x \geq 4.21 \text{ (correct to 3 sig. fig.)}$$

$$\text{So, the minimum value of } x = 4.21. \quad (1A)$$

8. Solution

$$(a) \quad \text{Interest obtained from simple interest} = Pnr\% = \frac{Pnr}{100}. \quad (1A)$$

$$\text{Interest obtained from compound interest} = P\left(1 + \frac{r\%}{12}\right)^{12n} - P \quad (1A)$$

$$= P\left[\left(1 + \frac{r}{1200}\right)^{12n} - 1\right]$$

$$\text{So, the required different} = P\left[\left(1 + \frac{r}{1200}\right)^{12n} - 1\right] - \frac{Pnr}{100}$$

$$= P\left[\left(1 + \frac{r}{1200}\right)^{12n} - \frac{nr}{100} - 1\right] \quad (1A)$$

(b) Substitute $n = 5$, $r = 12$ in the equation obtained from part a), we have

$$\$1560.21623 = P(1.01^{60} - 0.6 - 1) \quad (1M)$$

$$\Rightarrow P = \$7200 \quad (1A)$$

Section C - Long Questions (45 marks)

1. Solution

(a) $AE = EC = \frac{a}{2} \text{ cm}$

$$\angle HEF = \angle CED \text{ (common)}$$

$$\angle EHF = \angle ECD = 90^\circ$$

$$\angle EFH = \angle EDC \text{ (corr. } \angle s \text{ } HG \parallel CD)$$

$$\text{Therefore, } \triangle EFH \sim \triangle EDC \text{ (AAA)}$$

$$\text{Consider } \frac{EH}{EC} = \frac{HF}{CD} \Rightarrow \frac{EC - HC}{EC} = \frac{CD - FG}{CD}$$

$$\Rightarrow \frac{\frac{a}{2} - GD}{\frac{a}{2}} = \frac{a - FG}{a} \quad (1M)$$

$$\Rightarrow a - 2GD = a - FG$$

$$\Rightarrow GD = \frac{FG}{2} \quad (1A)$$

In $\triangle FGD$, by Pythagoras's Theorem, we have $FG^2 + GD^2 = FD^2$, substitute $GD = \frac{FG}{2}$,

$$\Rightarrow FG^2 + \left(\frac{FG}{2}\right)^2 = FD^2 \quad (1M)$$

$$\Rightarrow \frac{5FG^2}{4} = FD^2$$

$$\Rightarrow FG = \frac{2FD}{\sqrt{5}} \quad (1A)$$

(b) Consider $\triangle FGD$, by Pythagoras's Theorem, we have $FG^2 + GB^2 = BF^2$, from part a), we have $FG = \frac{2FD}{\sqrt{5}}$. Then, we obtain

$$\left(\frac{2FD}{\sqrt{5}}\right)^2 + GB^2 = BF^2 \Rightarrow \frac{4FD^2}{5} + GB^2 = BF^2 \text{ Divide both sides by } GB^2 \quad (1M)$$

$$\Rightarrow \frac{4}{5} \cdot \left(\frac{FD}{GB}\right)^2 + 1 = \left(\frac{BF}{FG}\right)^2$$

$$\Rightarrow \left(\frac{BF}{FG}\right)^2 = \frac{4}{5} \cdot \left(\frac{5}{4}\right)^2 + 1 \quad (1M)$$

$$\Rightarrow \left(\frac{BF}{FG}\right)^2 = \frac{9}{4} \quad (1A)$$

2. Solution

$$(a) \quad x^2 + \frac{1}{x^2} = \left(x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} \right) - 2$$

$$= \left(x + \frac{1}{x} \right)^2 - 2 \quad (1M)$$

$$= m^2 - 2 \quad (1A)$$

$$(b) \quad \text{Firstly, } x - \frac{1}{x} = \sqrt{\left(x - \frac{1}{x} \right)^2}$$

$$= \sqrt{x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}}$$

$$= \sqrt{x^2 + \frac{1}{x^2} - 2}$$

$$= \sqrt{m^2 - 4}$$

$$= \sqrt{(m+2)(m-2)} \quad (1A)$$

$$\text{Then, } x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x} \right) \left(x^2 + 1 + \frac{1}{x^2} \right)$$

$$= (m^2 - 1) \sqrt{(m+2)(m-2)}$$

$$= (m+1)(m-1) \sqrt{(m+2)(m-2)} \quad (1A)$$

(1M Either One)

(1M Either One)

3. Solution

$$(a)(i) \quad \text{Arc } AB = 2\pi r \frac{80^\circ}{360^\circ} = \frac{4\pi r}{9} \text{ cm}$$

Let the radius and the height of the base of the cone be r_1 and h respectively.

$$\text{Consider } 2\pi r_1 = \frac{4\pi r}{9}$$

$$r_1 = \frac{2r}{9} \text{ cm} \quad (1A)$$

By Pythagoras Theorem, we have

$$r_1^2 + h^2 = AC^2 \implies h^2 = r^2 - r_1^2$$

$$\implies h = \sqrt{r^2 - \left(\frac{2r}{9}\right)^2}$$

$$\implies h = \sqrt{\frac{77r^2}{81}} = \frac{r}{9}\sqrt{77} \text{ cm} \quad (1A)$$

$$\text{Hence, the required volume} = \frac{1}{3}\pi r_1^2 h \quad (1M)$$

$$= \frac{\pi}{3} \cdot \frac{4r^2}{81} \cdot \frac{r}{9}\sqrt{77}$$

$$= \frac{4\pi r^3 \sqrt{77}}{2187} \text{ cm}^3 \quad (1A)$$

$$(a)(ii) \quad \text{The required percentage change} = \frac{(0.85r)^3 - r^3}{r} \times 100\% \quad (1M)$$

$$= -38.5875\% \quad (\text{OR}, -38.6\%) \quad (1A)$$

(a)(iii) Let the radius of the base of the smaller cone be r_2 .

$$\text{By the ratio of similar triangles, we have } \frac{r_2}{r_1} = \frac{0.2r}{h} \quad (1M)$$

$$\implies r_2 = 0.2r \cdot \frac{2r}{9} \cdot \frac{9}{r\sqrt{77}}$$

$$\implies r_2 = \frac{2r}{5\sqrt{77}} \text{ cm} \quad (1A)$$

$$\text{Volume of the smaller cone} = \frac{1}{3}\pi r_2^2 (0.2r) = \frac{1}{3}\pi \frac{4r^2}{1925} (0.2r)$$

$$= \frac{4\pi r^3}{28875} \text{ cm}^3 \quad (1A)$$

$$\text{Hence, the volume of the remaining frustum} = \frac{4\pi r^3 \sqrt{77}}{2187} - \frac{4\pi r^3}{28875} \text{ cm}^3$$

$$= 4\pi r^3 \left(\frac{\sqrt{77}}{2187} - \frac{1}{28875} \right) \text{ cm}^3 \quad (1A)$$

4. Solution

$$(a)(i) \quad \text{Amount} = \$P \left(1 + \frac{10\%}{2} \right)^2 = \$1.05^2 P \quad (1A)$$

$$(a)(ii) \quad \text{Amount} = \$1.05^2 P \left(1 + \frac{10\%}{2} \right)^2 + \$(1 + 25\%)P \left(1 + \frac{10\%}{2} \right)^2 \quad (1M)$$

$$= \$(1.05^4 P + 1.25 \cdot 1.05^2 P) \quad (1A)$$

$$(a)(iii) \quad \text{Amount} = \$[1.05^4 P + 1.25 \cdot 1.05^2 P + 1.25P \cdot (1 + 25\%)] \left(1 + \frac{10\%}{2} \right)^2 \quad (1M)$$

$$= \$(1.05^6 P + 1.25 \cdot 1.05^4 P + \$1.25^2 \cdot 1.05^2 P) \quad (1A)$$

$$(b) \quad \begin{aligned} \text{Amount} &= 1.05^{40} P + 1.25 \cdot 1.05^{38} P + 1.25^2 \cdot 1.05^{36} P + \dots + 1.25^{18} \cdot 1.05^4 P + 1.25^{19} \cdot 1.05^2 P \quad (1A) \\ &= 1.05^2 P (1.05^{20} + 1.25 \cdot 1.05^{19} + 1.25^2 \cdot 1.05^{18} + \dots + 1.25^{18} \cdot 1.05^2 + 1.25^{19} \cdot 1.05) \end{aligned}$$

$$= 1.05^2 P \cdot 1.25^{20} \left[\left(\frac{1.05}{1.25} \right)^{20} + \left(\frac{1.05}{1.25} \right)^{19} + \left(\frac{1.05}{1.25} \right)^{18} + \dots + \left(\frac{1.05}{1.25} \right)^2 + \left(\frac{1.05}{1.25} \right) \right] \quad (1M)$$

$$= 1.05^2 P \cdot 1.25^{20} \cdot \frac{\frac{1.05}{1.25} \left(1 - \frac{1.05^{20}}{1.25^{20}} \right)}{1 - \frac{1.05}{1.25}} \quad (\text{using the given formula}) \quad (1M)$$

$$= \$5.25 \times 1.05^2 P (1.25^{20} - 1.05^{20}) \quad (1A)$$

$$(c) \quad \text{Consider } 5.25 \times 1.05^2 P (1.25^{20} - 1.05^{20}) \geq 27000000 \quad (1M)$$

$$\implies P \geq \$5547.768165$$

$$\text{So, the minimum value of } P = \$5548. \quad (1A)$$

5. Solution

(a)(i) Since,

$$CD = BD \quad (\text{sides of square } ABDC)$$

$$DE = DG \quad (\text{sides of square } EFGD)$$

$$\angle CDE = \angle CDB + \angle BDE = 90^\circ + \alpha$$

$$\angle BDG = \angle BDE + \angle EDG = 90^\circ + \alpha$$

$$= \angle CDE$$

Therefore, $\triangle CDE \cong \triangle BDG$ (SAS)

	No or Totally Incorrect Reasons	Incomplete or Partially Correct Reasons	Complete and Totally Correct Reasons
Score(s) Awarded	0 mark	1 or 2 mark(s)	3 marks

Marking Criteria

- Zero mark will be given if candidate CANNOT prove $\triangle CDE \cong \triangle BDG$ successfully.
- 2-3 marks will be given only if the rules for proving $\triangle CDE \cong \triangle BDG$ is correct (eg. SAS).
- 1 mark will be given if no reasons are shown but candidate can demonstrate effective logical flow.
- 3 marks will be given only if complete and totally correct reasons are shown.

(a)(ii) $\angle CED = \angle BGD = \beta$ (corr. $\angle$ s, $\triangle CDE \cong \triangle BDG$)

Then, $\angle ECD + \angle CDE + \angle CED = 180^\circ$ ($\angle$ sum of $\triangle$)

$$\Rightarrow \angle ECD = 180^\circ - 90^\circ - \alpha - \beta$$

$$= 90^\circ - (\alpha + \beta)$$

(1A)

Let the length of the altitude of CE that passes through D be L . We have

$$\begin{cases} L = CD \sin \angle ECD \\ L = ED \sin \angle CED \end{cases} \quad (1M)$$

$$\Rightarrow CD \sin \angle ECD = ED \sin \angle CED$$

$$\Rightarrow \frac{CD}{\sin \angle CED} = \frac{ED}{\sin \angle ECD}$$

$$\Rightarrow \frac{CD}{\sin \beta} = \frac{ED}{\sin [90^\circ - (\alpha + \beta)]}$$

$$\Rightarrow \frac{CD}{\sin \beta} = \frac{ED}{\cos(\alpha + \beta)} \quad (1A)$$

(b)(i) $\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \quad (1M)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \quad (1A)$$

Solution (Continue)

$$(b)(ii) \quad \angle CDG = 165^\circ \implies \text{reflex} \angle CDG = 360^\circ - 165^\circ = 195^\circ$$

$$\implies \alpha = 195^\circ - 2 \times 90^\circ$$

$$\implies \alpha = 15^\circ$$

(1A)

$$\angle GBD = \angle ECD = 90^\circ - (\alpha + \beta) \quad (\text{corr. } \angle s, \triangle CDE \cong \triangle BDG)$$

$$\angle ABG = 105^\circ \implies \angle ABD + \angle GBD = 105^\circ$$

$$\implies 90^\circ - (\alpha + \beta) = 105^\circ - 90^\circ$$

$$\implies 15^\circ + \beta = 75^\circ$$

$$\implies \beta = 60^\circ$$

(1A)

Then, consider $\frac{CD}{\sin \beta} = \frac{ED}{\cos(\alpha + \beta)}$

$$\implies \frac{CD}{ED} = \frac{\sin \beta}{\cos(\alpha + \beta)}$$

$$\implies \left(\frac{CD}{ED} \right)^2 = \left[\frac{\sin 60^\circ}{\cos(15^\circ + 60^\circ)} \right]^2 = \frac{\sin^2 60^\circ}{\cos^2 75^\circ} \quad (1M)$$

$$\implies \frac{\text{Area of } ABDC}{\text{Area of } EFGD} = \frac{3}{4} \cdot \frac{8}{3 - 2\sqrt{3} + 1} = \frac{3}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$\implies \frac{\text{Area of } ABDC}{\text{Area of } EFGD} = 3(2 + \sqrt{3}) \quad (1A)$$

— End of Paper —