Secondary 4 — Logarithmic Functions Final Exam Revision Exercise Solutions

1. (a)
$$\begin{aligned}\log 175 &= \log(25 \times 7)\\ &= \log 25 + \log 7\\ &= 2\log 5 + \log 7\\ &= 2a + b\end{aligned}$$

(b)
$$\begin{aligned}\log \sqrt{245} &= \frac{1}{2}\log(5 \times 7^2)\\ &= \frac{1}{2}\log 5 + \frac{1}{2}\cdot 2\log 7\\ &= \frac{1}{2}a + b\end{aligned}$$

(c)
$$\begin{aligned}\log \sqrt[3]{3.5} &= \frac{1}{3}\log\frac{5 \times 7}{10}\\ &= \frac{1}{3}(\log 5 + \log 7 - \log 10)\\ &= \frac{1}{3}(a + b - 1)\end{aligned}$$

2. (a)
$$\begin{aligned}\log 6 &= \log\frac{3 \times 20}{10}\\ &= \log 3 + \log 20 - \log 10\\ &= a + b - 1\end{aligned}$$

(a)
$$\begin{aligned}\log \sqrt{15} &= \frac{1}{2}\log\frac{3 \times 100}{20}\\ &= \frac{1}{3}(\log 3 + \log 100 - \log 20)\\ &= \frac{1}{3}(a - b + 2)\end{aligned}$$

3. (a)
$$\begin{aligned}\frac{5\log x - 3\log\sqrt{x}}{\log\frac{1}{x^3}} &= \frac{5\log x - \frac{3}{2}\log x}{-3\log x}\\ &= \frac{\frac{7}{2}\log x}{-3\log x}\\ &= -\frac{7}{6}\end{aligned}$$

3. (b)
$$\frac{4\log\frac{1}{\sqrt{x}}-\log\sqrt[3]{x^2}}{\log\left(x^3y^2\right)+2\log\frac{x}{y}}=\frac{4\cdot\left(-\frac{1}{2}\log x\right)-\frac{2}{3}\log x}{3\log x+2\log y+2\log x-2\log y}$$
$$=\frac{-\frac{8}{3}\log x}{5\log x}$$
$$=-\frac{8}{15}$$

4. (a)
$$\log(3x-y)-\log(2x)=y$$
$$\log\frac{3x-y}{2x}=y$$
$$\frac{3x-y}{2x}=10^y$$
$$3x-y=2x\cdot 10^y$$
$$x\left(3-2\cdot 10^y\right)=y$$
$$x=\frac{y}{3-2\cdot 10^y}$$

(b)
$$\log_3(x+2y)-\log_3(x-y)=2$$
$$\log_3\frac{x+2y}{x-y}=2$$
$$\frac{x+2y}{x-y}=3^2$$
$$x+2y=9x-9y$$
$$x=\frac{11}{8}y$$

(c)
$$\log_2(y-2x)-2=2\log_4(x-y)$$
$$\log_2(y-2x)-2=\frac{2\log_2(x-y)}{\log_2 4}$$
$$\log_2(y-2x)-2=\frac{2\log_2(x-y)}{2}$$
$$\log_2(y-2x)-2=\log_2(x-y)$$
$$\log_2(y-2x)-\log_2(x-y)=2$$
$$\log_2\frac{y-2x}{x-y}=2$$
$$\frac{y-2x}{x-y}=2^2$$
$$y-2x=4x-4y$$
$$x=\frac{5}{6}y$$

5. (a)
$$\frac{\log_8 y^3 + \log_8 \sqrt{y}}{2\log_2 y} = \frac{\log_8(y^3\sqrt{y})}{\log_2 y^2}$$
$$= \frac{\frac{\log_2 y^{\frac{7}{2}}}{\log_2 8}}{\log_2 y^2}$$
$$= \frac{\frac{1}{3}\log_2 y^{\frac{7}{2}}}{\log_2 y^2}$$
$$= \frac{\frac{1}{3}\cdot\frac{7}{2}\log_2 y}{2\log_2 y}$$
$$= \frac{7}{12}$$

(b)
$$\frac{3\log x}{\log x^5 - \log_{\frac{1}{10}} x^2} = \frac{3\log x}{5\log x - 2\cdot\frac{\log x}{\log\frac{1}{10}}}$$
$$= \frac{3\log x}{5\log x - 2\cdot\frac{\log x}{(-1)}}$$
$$= \frac{3\log x}{5\log x + 2\log x}$$
$$= \frac{3\log x}{7\log x}$$
$$= \frac{3}{7}$$

6. (a)
$$\log(x+6) = \log x + \log 6$$
$$\log(x+6) - \log x = \log 6$$
$$\log\frac{x+6}{x} = \log 6$$
$$\frac{x+6}{x} = 6$$
$$x = \frac{6}{5}$$

6. (b) $\log_5(3x-1)-\log_5(x+1)-1=0$

$$\log_5\frac{3x-1}{x+1}=1$$

$$\frac{3x-1}{x+1}=5^1$$

$$3x-1=5x+5$$

$$x=-3 \qquad \text{(rejected as } 3x-1=-10<0\text{)}$$

Therefore, there is no solution.

(c) $\log(3-2x^2)=2\log(x+1)$

$$\log(3-2x^2)=\log(x+1)^2$$

$$3-2x^2=x^2+2x+1$$

$$3x^2+2x-2=0$$

$$x=\frac{-2\pm\sqrt{28}}{6}$$

$$x=\frac{-2\pm2\sqrt{7}}{6}$$

$$x=\frac{-1+\sqrt{7}}{3} \quad \text{or} \quad x=\frac{-1-\sqrt{7}}{3} \quad \text{(rejected)}$$

7. $$\begin{cases}\log(x-5y)=2\\ \log x-\log y=1\end{cases}$$

From (1), we have $x-5y=100$ — (3)

From (2), we have $x=10y$ — (4)

Substitute (4) into (3), we have $10y-5y=100$

$$y=20$$

Substitute $y=20$ into (2), we have $x=200$

8. $$5^{2x-3}=12^{1-3x}$$

$$(2x-3)\log 5=(1-3x)\log 12$$

$$2x\log 5-3\log 5=\log 12-3x\log 12$$

$$x(2\log 5+3\log 12)=\log 12+3\log 5$$

$$x=\frac{\log 12+3\log 5}{2\log 5+3\log 12}$$

$$x=0.6852 \qquad \text{(correct to 4 d.p.)}$$

9.
$$2(3^{x+2}) = 5^{x-1}$$
$$\log 2 + (x+2)\log 3 = (x-1)\log 5$$
$$\log 2 + x\log 3 + 2\log 3 = x\log 5 - \log 5$$
$$x(\log 3 - \log 5) = -\log 5 - \log 2 - 2\log 3$$
$$x = \frac{-\log 5 - \log 2 - 2\log 3}{\log 3 - \log 5}$$
$$x = 8.8089 \qquad \text{(correct to 4 d.p.)}$$

10.
$$16^x \cdot 3 + 4^{x-3} \cdot 5 = 6$$
$$3 \cdot 4^{2x} + \frac{5}{64} \cdot 4^x - 6 = 0$$
Let $u = 4^x$, we have
$$3u^2 + \frac{5}{64}u - 6 = 0$$
$u = 1.40125267$ or $u = -1.427294337$ (rejected as $4^x > 0$ for all values of x)
$$4^x = 1.40125267$$
$$x = \frac{\log 1.40125267}{\log 4}$$
$$x = 0.2434 \qquad \text{(correct to 4 d.p.)}$$

11. Let t be the number of years required.
$$700000(1-15\%)^t < 400000$$
$$0.85^t < \frac{4}{7}$$
$$t \log 0.85 < \log \frac{4}{7}$$
$$t > \frac{\log \frac{4}{7}}{\log 0.85}$$
$$t > 3.44$$
Therefore, it takes at least 4 years for the value of the car to depreciate to less than \$400 000 .

12. Let t be the number of years required.

(a)
$$20000\left(1 + \frac{6\%}{2}\right)^{2t} - 20000 > 15000$$
$$1.03^{2t} > 1.75$$
$$t > 9.46$$
Therefore, it takes at least 10 years for him to get an interest more than \$15 000 .

12. (b) $20000\left(1+\frac{6\%}{4}\right)^{4t}-20000>15000$

$$1.015^{4t}>1.75$$

$$t>9.40$$

Therefore, it takes at least 10 years for him to get an interest more than \$15 000 .

13. (a) Substitute $(-1,\ 2.5)$ and $(3,\ 8.5)$ into $\log y=ax+b$. We have

$$\begin{cases}2.5=-a+b\\8.5=3a+b\end{cases}$$

Solving the system of equations, we have

$$a=\frac{3}{2}\quad\text{and}\quad b=4\,.$$

(b) From (a), we have $\log y=\frac{3}{2}x+4$.

Hence,

$$y=10^{\frac{3}{2}x+4}$$

$$y=10000\cdot10^{\frac{3}{2}x}$$

(c) When $y=\frac{1}{50}$, we have

$$\frac{1}{50}=10000\cdot10^{\frac{3}{2}x}$$

$$10^{\frac{3}{2}x}=\frac{1}{500000}$$

$$\frac{3}{2}x=\log\frac{1}{500000}$$

$$x=-3.799313336$$

$$x\approx-3.80\qquad\text{(corr. to 3 sig. fig.)}$$

14. (a) Let $\log_2 y = a \log_2 x + b$ be the graph of straight line, where a and b are constants.

Substitute $(2, -1)$ and $(4, 0)$ into the relationship.

We have $\begin{cases} -1 = 2a + b \\ 0 = 4a + b \end{cases}$

Solving the system of equations, we will obtain

$$a = \frac{1}{2} \quad \text{and} \quad b = -2$$

Hence, we have $\log_2 y = \frac{1}{2} \log_2 x - 2$

$$y = 2^{\log_2 \sqrt{x} - 2}$$

$$y = \frac{\sqrt{x}}{4}$$

(b) Required percentage change is

$$= \frac{\frac{\sqrt{1.69x}}{4} - \frac{\sqrt{x}}{4}}{\frac{\sqrt{x}}{4}} \times 100\%$$

$$= \frac{1.3 - 1}{1} \times 100\%$$

$$= 30\%$$

MC Full Solutions

1. **B**

<u>*Solution*</u>

$$\log 0.36 = \log \frac{3^2 \times 2^2}{100}$$

$$= 2\log 3 + 2\log 2 - \log 100$$

$$= 2m + 2n - 2$$

$$= 2(m + n - 1)$$

2. **C**

<u>*Solution*</u>

$$3^{2x} = 2^{3y}$$

$$2x \log 3 = 3y \log 2$$

$$\frac{x}{y} = \frac{3\log 2}{2\log 3}$$

$$= \frac{\log 8}{\log 9}$$

So, $x : y = \frac{\log 8}{\log 9}$

3. **C**

Solution

$\log_2(x-a)=3$

$x-a=2^3$

$x=8+a$

4. **A**

Solution

$\log_9 x^3=\log_3 x-1$

$\dfrac{\log_3 x^3}{\log_3 9}-\log_3 x=-1$

$\dfrac{1}{2}\log_3 x^3-\log_3 x=-1$

$\dfrac{3}{2}\log_3 x-\log_3 x=-1$

$\log_3 x=-2$

$x=3^{-2}$

$x=\dfrac{1}{9}$

5. **A**

Solution

Given that $10=3^x$ and $15=3^y$, we have

$x\log 3=1$

$\log 3=\dfrac{1}{x}$

Consider $\log 150=\log(10\times 15)$

$=\log(3^x\cdot 3^y)$

$=\log 3^{x+y}$

$=(x+y)\log 3$

$=(x+y)\dfrac{1}{x}$

$=1+\dfrac{y}{x}$

6. **B**

Solution

$\log_2 y+\log_{\frac{1}{2}} 2=3\log_2 x$

$\log_2 y-\log_2 2=3\log_2 x$

$\log_2\dfrac{y}{2}=\log_2 x^3$

$\dfrac{y}{2}=x^3$

$y=2x^3$

7. **B**

Solution

(1) is true:

As $a > 0$ and $b < 0$, therefore $ab < 0$.

Thus, $\log(ab)$ is undefined.

(2) is false:

If $a = 1$, we have $\log \frac{1}{a} = \log 1$

$= 0$, which is not a negative number.

(3) is true:

As $b < 0$, we have $\log_a k < 0$

$$\Longrightarrow \frac{\log k}{\log a} < 0$$

Since, $k > 1$, $\log k > 0$,

$$\Longrightarrow \frac{1}{\log a} < 0$$

$$\Longrightarrow \log a < 0$$

$$\Longrightarrow \log a < \log 1$$

$$\Longrightarrow a < 1$$

8. **A**

Solution

$$x = k y^n$$

$$\log x = \log k + \log y^n$$

$$n \log y = \log x - \log k$$

$$\log y = \frac{1}{n} \log x - \frac{\log k}{n}$$

Hence, the y-intercept is $-\frac{\log k}{n}$